Determination of the nonlinear elastic constants in a surface treated layer for aero-engine disk residual stress measurement

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Abstract. Existing ultrasonic stress evaluation methods utilize the acoustoelastic effect for bulk waves propagating in volume, which is unsuitable for a surface treated material, possessing a significant variation in material properties with depth. With knowledge of nonlinear elastic parameters – third-order elastic constants (TOEC) close to the surface of the sample, the acoustoelastic effect might be used with surface acoustic waves. This work is focused on the development of an independent method of TOEC measurement using the effect of nonlinear surface acoustic waves scattering – i.e. the effect of elastic waves interaction in a nonlinear medium.

In this paper, the possible three wave interactions of surface guided waves and bulk waves are described and formulae for the efficiency of harmonic generation and mode mixing are derived. A comparison of the efficiency of surface waves scattering in an isotropic medium for different interaction types is carried out with the help of nonlinear perturbation theory. First results for surface and bulk wave mixing with known second- and third-order elastic constants are shown.

Introduction

Ultrasonic methods provide viable tools for non-destructive evaluation of material properties, including external or residual stress. These methods for stress evaluation utilize the acoustoelastic effect of the dependency of the phase velocity of acoustic waves on (external and residual) stress [1]. The change in the wave velocity is linked to the stress by the material nonlinearity expressed in terms of acoustoelastic constants (AEC). These constants have to either be known from literature or by calibration with a known (e.g. applied) stress level. The acoustoelastic effect is widely used for volume inspection using bulk waves, which is as such not directly applicable to the evaluation of near surface residual stress.

A common method to enhance the fatigue life of components is the creation of compressive residual stress in the components' surface which suppresses the surface crack initiation and growth under (cyclic) tensile load. One major application is the surface treatment of blade discs in jet engines, fabricated of titanium and superalloys. These discs are exposed to high temperatures and in parallel to high centrifugal tensile stress. Up to the moment, evaluation of induced compressive stress amplitudes can only be done either during production by reference samples (Almen stripes) or with destructive (borehole drilling)/laboratory (x-ray) methods. Current airworthiness regulations call for an NDE tool for in-service control of residual stress. Otherwise the advantages of the surface treatment cannot be considered in the design [2].

The availability of a method to prove that the amount of compressive stress did not decrease during operation would allow a further weight optimization of jet engine elements, along with operational costs reduction and increased performance. Published work [3, 4, 5, 6] indicates, that surface layer stress estimation by the acoustoelastic effect might work but also reveals the challenges of that concept. One of the difficulties is that the surface wave velocity is not only influenced by the stress via the acoustoelastic effect but also directly by the microstructure changes due to plastic deformation. So we have (at least) two contributions, which have to be separated. Moreover, the first contribution depends on the AEC, representing the nonlinearity of the material, which is known to change significantly by plastic deformation [7], which is caused by the surface treatment to introduce the stresses. The main task of the present work is therefore to find an independent way to determine the nonlinearity of the surface layer that underwent plastic deformation at the sample itself, as a prerequisite to capture the stress contribution to the velocity change correctly. To achieve this goal, we suggest to make use of nonlinear propagation effects as alternatives to the acoustoelastic effect, namely harmonic generation and nonlinear wave mixing, including non-collinear wave mixing processes. The concept of the present work is to use these interactions to determine the third-order elastic constants (TOEC), which are linked with the sought AEC [8]. As a first step we shall solve the forward problem of predicting the harmonic and mixed wave generation efficiency for known TOEC. In this paper the current progress on the method development is reported. The possible interactions of surface guided waves and bulk waves with low inclination angles are described and formulae for the efficiency of harmonic and mixing mode generation are derived. First results of surface acoustic waves (SAW) and bulk waves mixing modeling for known second order elastic constants (SOEC) and TOECs are presented.

1. Concept

For isotropic elastic media, the potential energy density may be expanded in powers of three strain invariants I_1, I_2, I_3 [9,11]:

$$\Phi = \frac{1}{2}\lambda I_1^2 + \mu I_2 + \frac{1}{6}\alpha I_1^3 + \beta I_1 I_2 + \frac{4}{3}\gamma I_3 + \dots \quad (1)$$

where λ and μ are the second-order Lamé constants, α, β, γ are third-order Lamé constants (TOEC) [10], which may be expressed in terms of the Landau-Lifshitz constants *A*, *B*, *C* [9] or the Murnaghan constants *l*, *m*, *n* [11].

Within the cubic approximation of the strain energy, two input waves with frequencies ω_1 and ω_2 give rise to scattered waves of mixed frequency $n_1\omega_1 + n_2\omega_2$, where n_1 and n_2 are integers. This effect has been described theoretically [12, 13, 14] and observed experimentally [15, 16, 17].

One of the most well-known practical examples of wave scattering is the generation of the second harmonic along the propagation path in nonlinear media [18, 19, 20], which can be interpreted as interaction of two equal collinear waves. The process can be characterized by the acoustic nonlinearity parameter β (ANP), usually proportional to the ratio of the second harmonic amplitude to the square of the fundamental harmonic amplitude. ANP measurements in a nonlinear medium can also be utilized for surface layer NDE [20], as the ANP is related with the treatment intensity of the surface.

Available research results, related to practical applications of non-collinear wave scattering in NDE (i.e. works of Korneev and Demchenko [17, 22]), consider the interaction of bulk waves. For evaluation of material properties in a thin sub-surface layer one may also utilize surface acoustic waves (SAW or Rayleigh waves). This kind of waves propagates in a surface layer with effective penetration depth comparable to the wavelength [21]. To our knowledge, the use of Rayleigh wave mixing for elastic constants characterization of isotropic materials is not widely known.

The type (longitudinal, shear, SAW) and intensity of scattered wave is dependent on the types and intensities of the input waves, their frequencies, relative orientation of wave-vectors and polarization vectors, and on the material constants. Therefore, measurements of the scattering process in a surface layer can provide the information on the TOEC, necessary for stress characterization within the acoustoelastic effect.

2. Theoretical description

To get significant mixing intensity in the nonlinear interactions of longitudinal, shear and Rayleigh waves they must fulfill the phase matching (or resonance) conditions, which correspond to the energy and momentum conservation conditions in anharmonic phonon-phonon interaction [23]:

$$\omega_3 = \omega_1 \pm \omega_2 \quad (2,a)$$
$$\overrightarrow{k_3} = \overrightarrow{k_1} \pm \overrightarrow{k_2} \quad (2,b)$$

Analysis of allowed interactions and scattered amplitude intensities for bulk waves was introduced first by Jones and Kobett [14]. Later Korneev et al. took the polarization restrictions into account [22].

For the bulk wave - surface wave interactions in isotropic solids the only available reference is [24]. Allowed interactions are schematically shown in Fig. 1, where straight and curly lines represent bulk and Rayleigh waves, black and orange color stands for initial and scattered waves. Interaction processes of other combinations, involving surface acoustic waves, are forbidden because of the relative size of the phase velocities that enter Eq. 2, b.

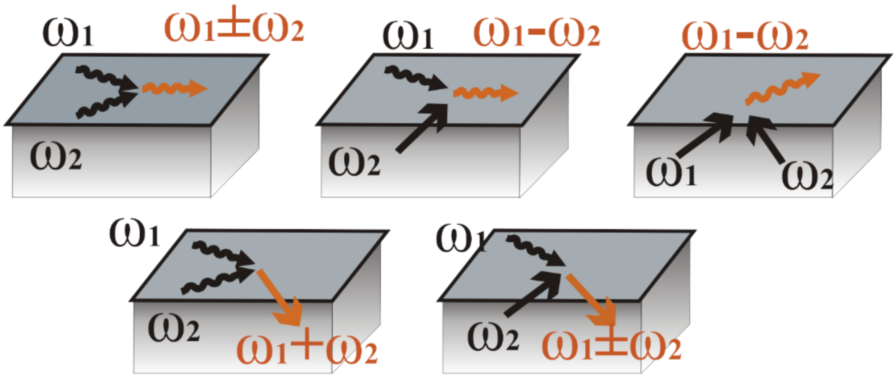

Figure 1: Available scattering modes for acoustic waves involving SAW [24]. Wavy lines: Rayleigh waves, straight lines: acoustic bulk waves, black: input waves, orange: scattered waves due to 2nd-order nonlinearity.

For determination of three TOECs in isotropic solids at least three independent measurements have to be performed. To identify the geometries with maximal scattering efficiency, the intensities of waves generated in nonlinear scattering processes for all allowed incoming and scattered wave combinations should be calculated.

As the numerical simulations of 3D wavefields in the time-domain for nonlinear isotropic media is computationally expensive, an approximate analytical solution for the scattered wave intensity [25] was implemented in Wolfram Mathematica 11.2 and solved for relevant interaction cases. This analytical solution uses perturbation theory and relies on the following assumptions:

1. The nonlinear perturbation is small compared to the linear part of the displacement.
2. The involved wavelengths are small compared to the sample thickness.
3. The medium is homogeneous and isotropic containing a uniform stress distribution.

While assumption 1 is clearly justified in practical cases and so is assumption 2 as long as the frequencies of the ultrasonic waves are sufficiently high, this does not need to be the case for assumption 3. This aspect will be further discussed below.

The equation of motion for the displacement field *u* can be presented in the following way:

$$\rho\, \ddot{u}_j(\vec{X},t) = T_{Lj,L}(\vec{X},t), \tag{3}$$

Where

$$T_{Lj} = \frac{\partial \Phi}{\partial u_{j,L}} = S_{jL} + S_{jLmM} u_{m,M} + \frac{1}{2} S_{jLmMnN} u_{m,M} u_{n,N} + \ldots \tag{4}$$

is the first Piola-Kirchhoff stress tensor, ρ is the mass density and the tensors *S* are defined in [26]. It is straightforward to express the quantities S_{jLmMnN} as linear combinations of second- and third-order elastic moduli λ, μ, α, β, γ.

Let the displacement field of the two input waves be of the form:

$$\vec{u}\big(\vec{X},t\big) = e^{i(\vec{q}\cdot\vec{X}-\omega t)} \vec{w}(Z|\vec{q}) + c.c\,, \tag{5}$$

where $\vec{X}$ - coordinate vector, $\vec{q}$ – 2D wave vector, and axis OZ is oriented normal to surface (the medium fills the halfspace Z < 0). Functions $\vec{w}(Z|\vec{q})$ are displacement profiles. For bulk waves as input waves, $\vec{q}$ is the projection of the 3D wave vector onto the surface plane (the OXY plane).

All of the analytical expressions and simulation results use the notation shown in Fig. 2. Wave vectors of incident waves ($\overrightarrow{q_1}$ and $\overrightarrow{q_2}$) are colored with black and oriented with angles α_1 and α_2 towards $\overrightarrow{OX}$ and the wave vector $\overrightarrow{q_3}$ of the scattered wave is colored with orange and points into the $\overrightarrow{OX}$ direction. For Rayleigh waves in isotropic media, the depth profile can be written as a sum of two exponentials:

$$\vec{w}(Z|\vec{q}) = \sum_{r=1}^{2} c(r) \begin{pmatrix} 1 \\ 0 \\ p(r) \end{pmatrix} e^{\alpha(r)Z}\,, \tag{6}$$

where coefficients $c(r)$, $p(r)$ and $\alpha(r)$ are defined in [25]. For longitudinal and shear waves expressions for $\vec{w}(Z|\vec{q})$ can be found in [21].

If the resonance conditions of Eq. 2 are fulfilled and the scattered wave type is a SAW, the solution for the displacement field of the scattered wave can be found in the form

$$\overrightarrow{u_3}\big(\vec{X},t\big) = e^{i(\overrightarrow{q_3}\cdot\vec{R}-\omega_3 t)} \vec{w}(Z|\overrightarrow{q_3})(-i)hX + \zeta(Z) + c.c\,, \tag{7}$$

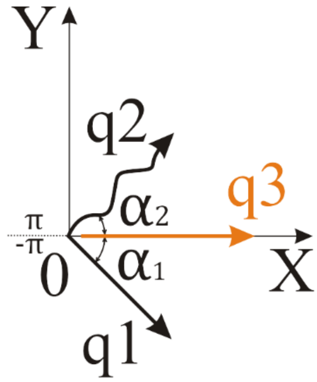

Figure 2: Coordinate system and definition of angles
(α defined in range between -*π* and *π* in OXY plane)

where $\vec{R}$ is the projection of $\vec{X}$ on the OXY plane. By inserting Eq. 7 into the equation of motion, projecting on $\vec{w}(Z|\vec{q})$, integrating by parts and making use of the nonlinear boundary conditions at the surface one obtains the following expression for the efficiency h of the Rayleigh wave generation through nonlinear interaction of two input waves:

$$h = \frac{\omega_3}{S_R} S_{jLmMnN} \int_{-\infty}^{0} [\hat{Q}_L\ (\overrightarrow{q_3}) w_l(Z|\overrightarrow{q_3})]^* [\hat{Q}_M(\overrightarrow{q_2}) w_m(Z|\overrightarrow{q_2})]\ [\hat{Q}_N\ (\overrightarrow{q_1}) w_n(Z|\overrightarrow{q_1})] dZ. \quad (8)$$

If the output wave is a bulk wave with 3D wave vector K = ($\overrightarrow{q_3}$,-k), it can be described as:

$$\overrightarrow{u_3}(\vec{X}, t) = e^{i(\overrightarrow{q_3} \cdot \vec{R} - \omega_3 t)} (B\overrightarrow{W} e^{-ikZ} + \vec{\zeta}(Z)) + c.c, \quad (9)$$

where zeta represents additional partial waves to satisfy the boundary conditions at the surface, and the solution for H (component of complex amplitude B of scattered bulk wave: B $= HA_1A_2$), can be calculated as an overlap integral similar to h in eq. (8), involving the displacement fields of the three nonlinearly interacting waves [25]. S_R depends on the normalization of *w* and on the Rayleigh wave velocity [25], $\overrightarrow{W}$ is a polarization unit vector and $\hat{Q}_L = \delta_{L1} i q_1 + \delta_{L2} i q_2 + \delta_{L3} \frac{\partial}{\partial Z}$. Therefore, the scattered wave efficiency can be expressed through depth profiles, polarization and wave vector components of initial waves. Based on this model, the following procedure has been implemented:

- Interaction case selection
- For each point on an (ω_1, ω_2) frequency grid:
  - Search for permitted initial waves orientation on a surface for a given scattered wave direction (Eq. 2)
  - Calculation of scattering efficiency for a given parameter combination
- Data storage, visual layout

Details of simulation are described in the following section.

3. Surface wave mixing modeling results

The forward problem (modeling of scattered wave efficiency with known material parameters) has been investigated for the nickel-base superalloy In718 *(*$\rho = 8.91 \frac{g}{cm^3}$, *B* = 199,94 *GPa (bulk modulus), v* = 0.294, TOEC values [6]: *l* = -520 *GPa*, *m* = -600 *GPa*, *n* = -480 *GPa).* In order to comply with the boundary conditions, the bulk waves were inclined 1.5° towards –OZ. Wave scattering modeling was preceded by several consistency checks. Comparison with [27] has been made for the case of collinear Rayleigh wave mixing.

Unfortunately, no published results for non-collinear wave scattering are known to the authors, which could have served as basis for a further check of our model.

As all of the possible α_1 and α_2 combinations are symmetrical with respect to the origin, we are expecting symmetry for scattered wave amplitudes for such coordinate pairs. In Fig. 3 a typical layout for scattering is presented as 3D plots with projections. Axes 1 and 2 ($\alpha_1$and α_2) form the plan view. The vertical axis is the relative scattering efficiency normalized by the scattered wave frequency ω_3. The frequency ratio is displayed by color-coding.

Simulation results for the case of two Rayleigh waves generating a longitudinal wave with sum-frequency are shown in Fig. 3 (in the following, the interactions would be labeled as /First input wave/ *[sign distinguishing sum and difference frequency generation]* /Second input wave/ → /Scattered wave/, i.e. RW+RW→P for the given example).

Simulation results are shown on Tab. 1 with the three most intensive interactions highlighted.

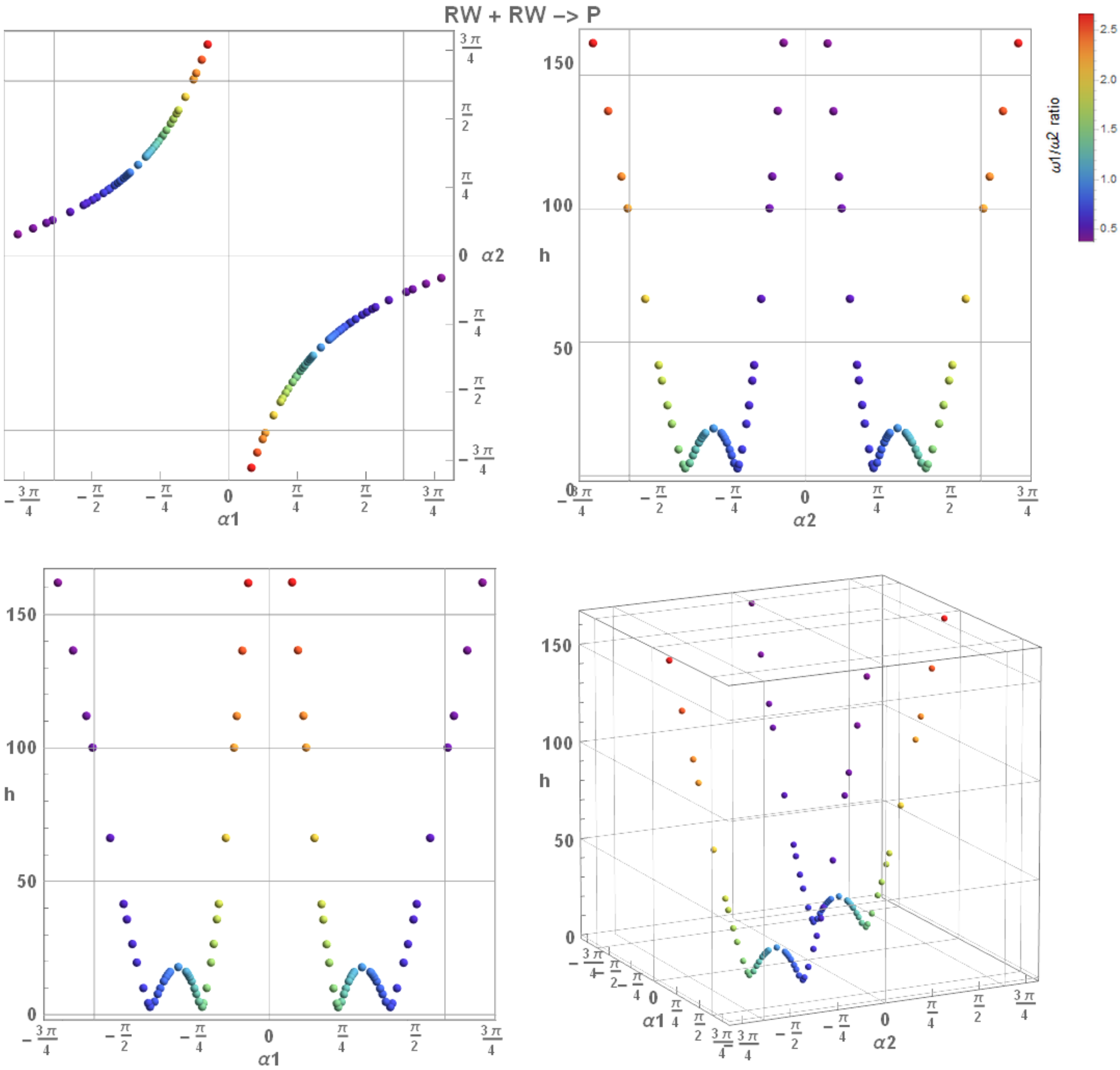

Figure 3: RW+RW→ P, efficiency H of the longitudinal wave generation by two Rayleigh waves, propagating at angles α_1 and α_2, with a frequency ratio ω_1/ω_2

Table 1: Scattering efficiency simulation results

Interaction case	α_1, rad	α_2, rad	Scattering efficiency H/ω_3 | h/ω_3	ω_1/ω_2
RW+RW→P	-2.42	0.25	162.23	0.38
RW+RW→SV	-1.34	0.1	60.86	0.1
RW+P→P	0.19	-2.4	70.38	1.8
RW+ P→SV	-0.02	0.3	6.17	6.
RW+SV→P	-2.96	0.05	180.41	0.29
RW+SV→SV	-0.09	1.33	75.71	10.
P-P→RW	0.96	1.14	24.19	1.11
P-SV→RW	2.96	3.02	72.82	1.29
SV-P→RW	-0.05	-0.88	22.99	9.
SV-SV→RW	-0.1	-1.34	227.83	10.

4. Discussion and further work

First results on anharmonic wave scattering simulations involving surface acoustic waves and bulk acoustic waves are presented. Geometrical layouts and a comparison of the relative efficiencies of the various scattering processes served as a basis for the design of experiments. Laser Doppler vibrometry and contact measurements are currently underway to verify the scattering model described above. The following step will be the investigation of the inversion problem: evaluation of third-order elastic constants by recording the intensity of the scattered wave as function of the parameters of the input waves. Subsequently, the model will be extended to allow for variations of the material constants along the coordinate axis normal to the surface, i.e. assumption 3 in section 2 will be relaxed.

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