Semi-analytical computation of a quasi-static field induced by a 3D eddy current probe in anisotropic material with parallel rough interfaces

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Abstract

This paper deals with the computation of a quasi-static field induced by a 3D eddy current coil in homogeneous and anisotropic material presenting a local deformation in the planar geometry. Using a semi-analytical approach, we resolve Maxwell's equations in the Fourier domain: the electromagnetic (EM) field is expanded as a sum of eigenmodes which are computed in the new curvilinear coordinate system presented by a new metric tensor. Analytically, the boundary conditions are applied to compute the unknown coefficients of the modal expansion. Numerical experiments are performed and simulation results are compared to FE data.

keywords: Anisotropy, Curvilinear Coordinate Method, Modal expension, Scattering metrix.

I. Introduction

The inspection of unidirectional composite material as CFRP for characterization or crack detection is in a great demand due to its excessive use in the industrial field. Despite the low conductivity along carbon fibers comparing to metallic structure, Eddy Current Non-Destructive Testing technique (ECNDT) has shown a great potential in detecting damages as delamination or fiber breakage. This non-homogeneous material presents high anisotropy and random positioning of the fibers which makes its interactions with EM waves is highly depended on the microstructure of the material itself. To overcome this modeling challenge, a prior phase of homogenization is considered so that the conductivities are constant along different axis.

In a previous work, a semi-analytical model based on a modal approach was developed for the fast computation of quasi-static field induced by a 3D EC probe in planar, stratified and anisotropic material. As an extension, a local geometrical deformation is introduced in the planar structure to model delamination. To address such configuration, we use the Curvilinear Coordinates Method (CCM) based on a translation to a new coordinate system which fits the deformation. Thus, Maxwell's equations are written in the covariant form and a new matric tensor is presented. This leads us to write in a simpler way the boundary conditions [1]. Basically, the numerical model consists in resolving Maxwell's equations in the Fourier domain: the EM field is expanded as sum of eigenmodes in each media. Applying BC, one can determine the unknown coefficients of the modal expansion. To adress the multilayer aspect, a stable and recursive scattering matrix algorithm is implemented, taking into account the wave propagation through the structure, the modal expansion in each media and BC at each interface. The simulated data are compared to FE results.

II. Methodology

A. Formalism

Let us consider a planar workpiece corrupted by a local deformation as displayed in Fig. 1, and expressed by means

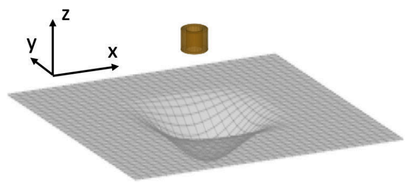

Fig. 1. Local 3D smooth deformation in the geometry

of an arbitrary 2D function $a(x, y)$.The new coordinate system is presented by:

$$\begin{cases} x^1 &= x \\ x^2 &= y \\ x^3 &= z - a(x,y) \end{cases} \tag{1}$$

Working in harmonic regime with $e^{i\omega t}$ time dependancy and a non-magnetic material ($\bar{\bar{\mu}} = \mu$), Maxwell's equations appear in the covariant form (Eq.2) with a new metric tensor $[g^{ij}]$ as presented in Eq.3.

$$\begin{cases} \partial_j B_j = 0 \\ \partial_j D_j = 0 \\ \xi^{ijk}\partial_j E_k = +j\omega B^i \\ \xi^{ijk}\partial_j H_k = J_p^i + \bar{\bar{\sigma}} E^i \end{cases} \quad and \quad \begin{cases} D^i = \bar{\bar{\varepsilon}}\sqrt{g} g^{ij} E_j \\ B^i = \mu\sqrt{g} g^{ij} H_j \\ J^i = \bar{\bar{\sigma}}\sqrt{g} g^{ij} E_j \end{cases} \tag{2}$$

where $g = det([g^{ij}])$ and

$$\bar{\bar{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}, \quad [g^{ij}] = \begin{bmatrix} 1 & 0 & -\frac{\partial a}{\partial x} \\ 0 & 1 & -\frac{\partial a}{\partial y} \\ -\frac{\partial a}{\partial x} & -\frac{\partial a}{\partial y} & 1 + \frac{\partial a}{\partial x} + \frac{\partial a}{\partial y} \end{bmatrix} \tag{3}$$

B. Numerical approach

1) **Modal exxpansion***:* The modal approach is based on a TE/TM decompostion in isotropic media with a polarizaion along x^3. The contravariant components E^3 and H^3 satisfy Helmhotz's equation translated to the new coordinate system.

$$[\partial_1 + \partial_2 + (1 + \dot{a}^2)\partial_3^2 - (\dot{a}\partial_1 + \partial_1 \dot{a})\partial_3 + k^2]\Psi = 0 \tag{4}$$

Translating Maxwell's equations in the Fourier domain, we assume a variable separation, $\psi(x^1, x^2, x^3) = \psi(x^1, x^2)\ \exp(-i\lambda x^3)$, so that the $\partial_3 = -i\gamma$. These two potential can be written as sum of modal functions [2]. returning to Maxwell's equations, we can reconstruct the covariant components.

However in anisotropic media, all the covariant components of the electrical field E_1, E_2 and the magnetic field H_1, H_2 are coupled and satisfy a full eigenvalues system to be solved leading to eigenmodes expansion.

$$\partial_3 \begin{bmatrix} E_1 \\ E_2 \\ H_1 \\ H_2 \end{bmatrix} = \mathcal{L} \begin{bmatrix} E_1 \\ E_2 \\ H_1 \\ H_2 \end{bmatrix} = [\mathcal{L}_{ij}] \begin{bmatrix} E_1 \\ E_2 \\ H_1 \\ H_2 \end{bmatrix} \quad (5)$$

Where $\mathcal{L}$ is a matrix of operators. e.g:

$$\mathcal{L}_{13} = -ik\mu g^{21} + ik\mu g^{23}[g^{33}]^{-1}g^{31} - \frac{\varepsilon_{zz}}{ik}\partial_1([g^{33}]^{-1}\partial_2) \quad (6)$$

Finally the contravariant components are reconstructed.

2) ***Boundary Conditions and Scattering-matrix****:* The BC at a separating interface are defined by the continuity of the covariant components of the EM field. Hence, the unknown coefficients of the modal expansion verify an inverse problem to be solved.

To tackle multilayer problem, we use a recursive scattering matrix (s-matrix) algorithm [3]. For a given structure as shown in Fig.2. we obtain two types of matrices:

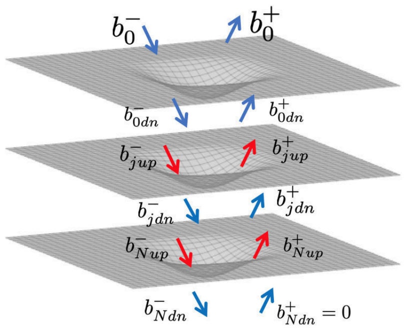

Fig. 2. Multilayered structure

- the interface matrix: takes into account the modal expansions above and under the considered interface and links the outputs and inputs by applying BC:

$$\begin{bmatrix} b_{jup}^+ \\ b_{jdn}^- \end{bmatrix} = \mathbf{S}_{inter} \begin{bmatrix} b_{jup}^- \\ b_{jdn}^+ \end{bmatrix} \quad (7)$$

- the layer matrix: takes into account the propagation of the field through a layer:

$$\begin{bmatrix} b_{(j-1)dn}^+ \\ b_{jup}^- \end{bmatrix} = \mathbf{S}_{layer} \begin{bmatrix} b_{(j-1)dn}^- \\ b_{jup}^+ \end{bmatrix} \quad (8)$$

Recursively, we concatinate these two types of s-matrices and one can obtain a global s-matrix presenting the whole structure as a black-box:

$$\begin{bmatrix} b_0^+ \\ b_{Ndn}^- \end{bmatrix} = \mathbf{S}_g \begin{bmatrix} b_{Ndn}^+ \\ b_0^- \end{bmatrix} \quad (9)$$

III. Results

We consider a 3D circular EC coil as presented in Fig.1, scanning anisotropic plate along the x direction from $x = -10$ mm to $x = 10$ mm with 1mm step. In Fig.3, we display on the left the EC density induced by the coil on the top surface $x^3 = 0$ at the position $x = 2$mm. The conductivity tensor is diagonal with $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}) = (10^6; 10^5; 10^5)$ S/m. On the right we present $\Re(\Delta Z)$ and $\Im(\Delta Z)$ variation along the scanning axis. the impedance variation is computed by means of Auld Formula [2]. These preliminary results are compared to FE data.

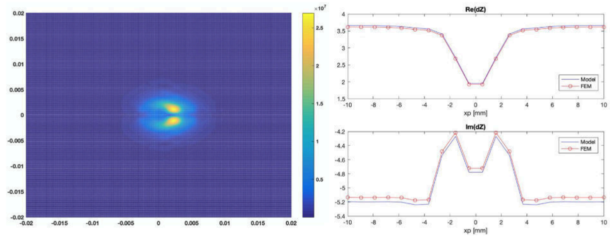

Fig. 3. ECD distribution on $x^3 = 0$ and ΔZ variation along x axis

We illustrate the computing time in TABLE I. the semi-analytical model shows a greate performance.

	Semi-analytical model	FEM
Computing Time (mn)	9 mn	135 mn

TABLE I
COMPUTING TIME COMPARISON

IV. Discussion

By applying 2D TF to Maxwell's equations, one can define the numbers of modes M_u and M_v along x^1 and x^2 as the trancature number of the field representation in the Fourier domain. So the greater these numbers are the better is the accuracy but it's time consuming. Contrarely to the planar case and from numerical point of view, the new matric tensor and the general form of the conductivity tensor generate a full matrices of size $(L = (2M_u+1)*(2M_v+1))$ to manipulate thus we are limited in the trancature. This hardware obtacle open perspective to optimization and parallel computing studies.

V. Conclusion

This paper presents a second part of PhD work in which we aim to characterize a composite material via EC inspection. we introduced a local deformation into the planar geometry to modal delamination. A semi-analytical model based on a modal expansion of the EM field was adopted for the resolution of Maxwell's equation in their covariant form in a new curvilinear coordiantes system.

VI. Aknowledgement

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