Electromagnetic modeling
Application to eddy current inspection of complex material
used in aeronautic industry

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CEA LIST
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Introduction:
Contribution within NDTonAir project

NDTonAir aims to

- Studying new physical phenomena and new sensors;
- Developing analytical and numerical models to correlate the results inspection with the properties of the materials;
- Quantifying NDT techniques by their probability of detecting defaults;
- Developing procedures for automatic detection and classification of cracks;
- Transferring these results to industry.
**Introduction:**

Contribution within NDTonAir project

We contribute in

- Studying new physical phenomena and new sensors;
- Developing analytical and numerical models to correlate the inspection results with the properties of the materials;
- Quantifying NDT techniques by their probability of detecting faults reference;
- Developing procedures for automatic detection and classification faults;
- Transferring these results to industry.
We are interested in developing a numerical model dedicated to the simulation of the interaction of an electromagnetic field produced by a coil with a complex structure as composite material.

In general, the principle of a model is to replace a complex system by a simple object or operator reproducing the main aspects or behaviours from the original.

Models very often use systems of partial differential equations (PDE) of which we do not know analytical solutions but instead we solve them numerically by transforming the continuous equations of physics into a discrete problem.

The different steps to model a complex system are:

- Development of a mesh or calculating grid: Discretization of the physics equations.
- Solving discrete equations (often linear systems to be solved).
- Computer transcription and programming of discreet relationships.
- Numerical simulation and exploitation of the results.
Introduction: Electromagnetic Modeling

- Electromagnetic testing methods use magnetism and electricity to detect and assess faults, fractures or corrosion.
- ET induced electric currents, magnetic fields, or both inside an object to be tested and observes the electromagnetic response of this object.
The electromagnetic methods (EM) include eddy current (EC) testing, Remote field testing (RFT), Magnetic flux leakage (MFL) and AC field measurement (ACFM).

In each of these techniques, the underlying physics is fundamentally different across domains.
Introduction: Electromagnetic Modeling

- The electromagnetic modeling consists on developing models based on mathematical and physical background.
- The most common used set of (PDE) describing the electromagnetic state of the system are Maxwell’s equations.

### Maxwell’s Equations: Differential and Integral Forms

<table>
<thead>
<tr>
<th>Name of Law</th>
<th>Differential Form</th>
<th>Integral Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss’s Law</td>
<td>( \nabla \cdot \mathbf{D} = \rho_v )</td>
<td>( \int_S \mathbf{D} \cdot d\mathbf{s} = \int_S \rho_v , dv = Q )</td>
</tr>
<tr>
<td>Faraday’s Law</td>
<td>( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} )</td>
<td>( \oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} )</td>
</tr>
<tr>
<td>Gauss’s Law of Magnetics</td>
<td>( \nabla \cdot \mathbf{B} = 0 )</td>
<td>( \oint_S \mathbf{B} \cdot d\mathbf{s} = 0 )</td>
</tr>
<tr>
<td>Ampere’s Law</td>
<td>( \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} )</td>
<td>( \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s} )</td>
</tr>
</tbody>
</table>
EC inspection simulation: Motivation

- The modeling of electromagnetic phenomena has become a major tool of NDT-EC, allowing industrialists to partially replace costly experimental studies necessary for designing new control processes (new sensor, new geometry of parts, sensitivity study, parametric study ...).

- In addition, it is a valuable aid in interpreting signals which is particularly delicate. Furthermore, it is clear that developing models fast and accurately is a fundamental step towards signal reversal control.
Analytical methods are often used for studying configurations with simple geometries. As example of analytical solutions for eddy current problems [Deads-68]:

\[ A(r, z) = \int_0^\infty [A(\alpha) \exp(\alpha_i z) + B(\alpha) \exp(-\alpha_i z)] [C(\alpha) J_1(\alpha r) + D(\alpha) Y_1(\alpha r)] \, d\alpha \]

[Theodoros Theodoulidis] presents the calculation analysis of the electromagnetic field and the impedance of a cylindrical coil with eddy currents arbitrarily oriented above half a space driver.

\[ Z = j\omega \pi \mu_0 L^2 \left\langle \chi(kr_1, kr_2) \right| \kappa^{-7} \left\{ 2(z_2 - z_1)\kappa - 2I \right. \\
+ 2e^{-k(z_2 - z_1)} + \left( e^{-kz_1} - e^{-kz_2} \right) \Gamma \\
\left. \left( e^{-kz_1} - e^{-kz_2} \right) \hat{\chi}(kr_1, kr_2) \right\rangle. \]
There are many numerical methods for simulating electromagnetic phenomena. The most famous one is the finite element method (MEF) which, since its introduction in the 1950s, have been considerably used and its field of application has widened to most fields of modern physics. Its main advantage lies in its generality which allows it to process configurations with geometric characteristics and very varied physical.
Semi-analytical methods are very effective for solving partial differential equations where part of the problem is solved numerically and the other part analytically.

Among these approaches, we cite the volume integral methods (VIM) and surface integral methods (SIM), which are based on an integral formulation of Maxwell's equations.
The VIM formulation implemented into CIVA for eddy current inspection is based on writing analytically the Green dyads function associated with the geometry of the configuration.

This method is very fast since only the defect is discretized in elementary cells.

First, the corresponding primary field is the field created by the probe in the absence of a fault is calculated.

Then, the disturbance of this primary field, induced by the presence of the fault volume, is obtained by super-imposing the fields radiated by the different cells forming the defect considered as a secondary source.

Finally, the response of the sensor to this disturbance is calculated.
Semi-analytical modeling of EC inspection:
EC inspection in CIVA
Resume of my work

Effective parameters
➢ Coil parameters
  • Outer / Inner diameter + Height
  • Number of turns and lift-off
  • Frequency and Current excitation

➢ Specimen parameters
  • Conductivity tensor
  • Permittivity tensor
  • Calculation area
  • Number of layer
  • Thickness

➢ Numerical parameters
  • Number of modes
  • Spatial resolution

Numerical Model: Semi-analytical approach
➢ Harmonic regime
  \( e^{\text{i} \omega t} \)
  time dependency

➢ Homogenized material constant \( \sigma \) along different directions

➢ Resolution in Fourier domain

\[
\nabla \times \mathbf{E} = -i\omega \mu \mathbf{H} \\
\n\nabla \times \mathbf{H} = i\omega \varepsilon \mathbf{E} + \sigma \mathbf{E}
\]

EM field
Coil response

\( \mathbf{E}_x, \mathbf{E}_y, \mathbf{E}_z \)
\( \mathbf{H}_x, \mathbf{H}_y, \mathbf{H}_z \)
Impedance

Diagnostics
Analysis
Resume of my work
Curvilinear Coordinate Method

- The C method was born in the 80's in the need to rigorously solve diffraction problems on surfaces wavy periodicals in resonance regime.

- The main difficulty of these problems is the application of the boundary conditions. It's obvious that any method to solve Maxwell's equation is more effective when it is adapted to the geometry of the problem.
Resume of my work
Curvilinear Coordinate Method

\[ \begin{align*}
\xi^{i,j,k} \partial_j E_k &= -i \omega \mu_0 \mu_p \sqrt{g} g^{ij} H_j \\
\xi^{i,j,k} \partial_j H_k &= i \omega \varepsilon_0 \varepsilon_p \sqrt{g} g^{ij} E_j
\end{align*} \]

\( \{ i, j, k \} \in \{1, 2, 3\} \)

Change of coordinates \( \rightarrow \) Translation

\[ \begin{align*}
x &= x^1 \\
y &= x^2 \\
z &= x^3 + a(x^1, x^2)
\end{align*} \]

New metric tensor

\[
(g^{ij}) = \begin{bmatrix}
1 & 0 & \dot{a}_1 \\
0 & 1 & \dot{a}_2 \\
-\dot{a}_1 & -\dot{a}_2 & 1 + \dot{a}_1^2 + \dot{a}_2^2
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & g^{13} \\
0 & 1 & g^{23} \\
g^{13} & g^{23} & g^{33}
\end{bmatrix}
\]

\[ \begin{align*}
\dot{a}_1 &= \partial_x a = \frac{\partial a}{\partial x} \\
\dot{a}_2 &= \partial_y a = \frac{\partial a}{\partial y}
\end{align*} \]
Tangential components versus longitudinal components: \( E_3, \quad H_3 \)

\[
(k_c^2 - \gamma^2) \begin{bmatrix} E_2 \\ G_1 \\ G_2 \\ E_1 \end{bmatrix} = \begin{bmatrix} +i k \mu_0 \mu_p (\partial_1 + g^{13} \partial_z) \\ \partial_1 \partial_3 - k_c^2 g^{13} \\ \partial_2 \partial_3 - k_c^2 g^{23} \\ -i k \mu_0 \mu_p (\partial_2 \phi + g^{23} \partial_3) \end{bmatrix} Z_0 H_3 + \begin{bmatrix} \partial_2 \partial_3 - k_c^2 g^{23} \\ +i k \varepsilon_0 \varepsilon_p (\partial_2 + g^{23} \partial_3 \phi) \\ -i k \varepsilon_0 \varepsilon_p (\partial_1 + g^{13} \partial_3 \phi) \\ \partial_1 \partial_3 - k_c^2 g^{13} \end{bmatrix} E_3
\]

TE / TM decomposition

\[
\begin{bmatrix} E_2 \\ Z_0 H_1 \\ Z_0 H_2 \\ E_1 \end{bmatrix} = \Gamma^{TE} \begin{bmatrix} \Phi^{TE} \end{bmatrix} \phi + \Gamma^{TM} \begin{bmatrix} \Phi^{TM} \end{bmatrix} \phi
\]

Covariant Helmholtz equations:

\[
[g^{33} \partial_3^2 + (\partial_1 g^{13} + g^{13} \partial_1) \partial_3 + (\partial_2 g^{23} + g^{23} \partial_2) \partial_3 + \partial_1^2 + \partial_2^2 + k_c^2] \phi = 0
\]

We introduce an auxiliary variable

\[
\phi'(x^1, x^2) = \partial_3 \phi(x^1, x^2) = -i \gamma \phi(x^1, x^2)
\]

We obtain an eigenvalue equation to solve

\[
\begin{bmatrix} i \phi' \\ \phi \end{bmatrix} \gamma = \begin{bmatrix} \mathcal{L}_p \end{bmatrix} \begin{bmatrix} i \phi' \\ \phi \end{bmatrix}
\]

By using Fourier basis functions

\[
\phi(x^1, x^2, x^3) = \mathcal{F}^{-1} \begin{bmatrix} \hat{\phi}(\alpha, \beta, x^3) \end{bmatrix} \Rightarrow [\hat{\phi}_{mn}] \begin{cases} m = -M, \ldots, M \\ n = -N, \ldots, +N \end{cases}
\]

Discretization and truncation

\[
g^{ij} \phi \rightarrow \left( \hat{g}^{ij} \ast \hat{\phi} \right)(\alpha, \beta, x^3) \Rightarrow [g] [\hat{\phi}_{mn}] \begin{bmatrix} \beta = diag(\beta) \end{bmatrix}
\]

\[
\partial_1 \phi(x^1, x^2, x^3) = \partial_2 \phi(x^1, x^2, x^3) \Rightarrow [D_1] [\hat{\phi}_{mn}] = -i [I_{2M+1} \otimes \alpha] [\hat{\phi}_{mn}]
\]

\[
\partial_1 \phi(x^1, x^2, x^3) - i \alpha \hat{\phi}(\alpha, \beta, x^3) \Rightarrow [D_2] [\hat{\phi}_{mn}] = -i [\beta \otimes I_{2N+1}] [\hat{\phi}_{mn}]
\]
All the EM field components are coupled.

More complex eigenvalue system to be resolve:

$$\bar{\sigma}, \bar{\mathcal{g}}$$ are Full matrices.

$$\partial_3 \begin{bmatrix} E_x \\ E_y \\ H_x \\ H_y \end{bmatrix} = [\mathcal{L}_{ij}] \begin{bmatrix} E_x \\ E_y \\ H_x \\ H_y \end{bmatrix}$$

E.g.,

$$\mathcal{L}_{11} = -g^{23}g^{33}^{-1}\partial_v - \partial_u[(\varepsilon_{xx}g^{13} + \varepsilon_{yx}g^{23})*g^{33}^{-1}]$$

$$\mathcal{L}_{32} = ik[\varepsilon_{yy} - \varepsilon_{zz}g^{23}*g^{33}^{-1}] + (\varepsilon_{xy}g^{13} + \varepsilon_{yy}g^{23}) - \frac{1}{ik\mu}\partial_u([g^{33}]^{-1}\partial_v)$$

$$\begin{bmatrix} E_t \\ H_t \end{bmatrix} = \Psi^- \Lambda^- + \Psi^+ \Lambda^+$$

Eigen modes sorted regarding the eigenvalues

Unknown coefficients to be determined using BC.
Scattering matrix method

\[
\begin{bmatrix}
  a_j^{+(j)} \\
  a_{j-1}^{-}(j+1)
\end{bmatrix} =
\begin{bmatrix}
  S_{11}^j & S_{12}^j \\
  S_{21}^j & S_{22}^j
\end{bmatrix}
\begin{bmatrix}
  a_j^{+(j+1)} \\
  a_{j-1}^{-}(j)
\end{bmatrix} = S_j
\begin{bmatrix}
  a_j^{+(j+1)} \\
  a_{j-1}^{-}(j)
\end{bmatrix}
\]

Interface matrix \( S_j = [\Psi_j^{+}_{j-1} - \Psi_j^{-}]^{-1} [\Psi_j^{+} - \Psi_j^{-}_{j-1}] \)

Layer matrix

\[
\begin{bmatrix}
  a_j^{+(j)} \\
  a_{j}^{-}(j+1)
\end{bmatrix} =
\begin{bmatrix}
  0 & P_j^+ \\
  P_j^- & 0
\end{bmatrix}
\begin{bmatrix}
  a_j^{-}(j) \\
  a_j^{+(j+1)}
\end{bmatrix} = S_{layer}^i
\begin{bmatrix}
  a_j^{-}(j) \\
  a_j^{+(j+1)}
\end{bmatrix}
\]

Non parallel interface

\[
P_{j^+} = [\Psi_j^{+}(j)]^{-1} \Psi_j^{+}(j+1) e^{i \gamma_j,q t_j(x^1)}
\]
\[
P_{j^-} = [\Psi_j^{-}(j+1)]^{-1} \Psi_j^{-}(j) e^{-i \gamma_j,q t_j(x^1)}
\]

Parallel interface

\[
P_{j^+} = \text{diag}_q(e^{i \gamma_j,q t_j})
\]
\[
P_{j^-} = \text{diag}_q(e^{-i \gamma_j,q t_j})
\]
Results

Eddy current distribution in each layer

Polar diagram of the impedance variation
Results

\[ a(x, y) = h_p a(x) a(y), \]

Not a restriction! Any function may be implemented!

\[ a(s) = \frac{1}{2} \left[ 1 + \cos \left(2\pi \frac{s}{L_s} \right) \right], \quad \forall s \in [-L_s/2, L_s/2], \]

\[ s = x \text{ or } y \]

<table>
<thead>
<tr>
<th>Plaque</th>
<th>((\sigma_t, \sigma_l, \sigma_n))</th>
<th>(1, 10^{-2}, 10^{-2}) SM/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>hauteur profils</td>
<td>(h_p)</td>
<td>-0.5 mm</td>
</tr>
<tr>
<td>Longueur défauts selon x</td>
<td>Lx</td>
<td>6 mm</td>
</tr>
<tr>
<td>Longueur défauts selon y</td>
<td>Ly</td>
<td>6 mm</td>
</tr>
</tbody>
</table>

\[
\bar{\sigma} = \begin{bmatrix}
\sigma_1 \cos^2(\theta) + \sigma_l \sin^2(\theta) & \frac{\sigma_1 - \sigma_l}{2} \sin(2\theta) & 0 \\
\frac{\sigma_1 - \sigma_l}{2} \sin(2\theta) & \sigma_l \sin^2(\theta) + \sigma_t \cos^2(\theta) & 0 \\
0 & 0 & \sigma_n
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>Probe</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>(f)</td>
</tr>
<tr>
<td>driving current</td>
<td>(I_0)</td>
</tr>
<tr>
<td>Internal radius</td>
<td>(r_{int})</td>
</tr>
<tr>
<td>External radius</td>
<td>(r_{ext})</td>
</tr>
<tr>
<td>Height</td>
<td>(H)</td>
</tr>
<tr>
<td>Number of turns</td>
<td>(N)</td>
</tr>
<tr>
<td>Lift-off</td>
<td>(l_0)</td>
</tr>
</tbody>
</table>

Last Training Event: 03-05 June 2020
Results
Conclusion: Perspectives & Possible coupling with other NDT techniques

- **Perspectives**
  - Extend the model to non-homogeneous material
  - Create a database with a big variety of complex deformation
  - Machine learning algorithm can be used to identify the characteristics of a detected crack

- **Coupling with NDT techniques**
  - Eddy current coils can be used as source of heat for thermography NDT technique
  - Eddy current coils can be used to produce acoustic waves through the material
  - Magneto-optic imaging combines the magneto-optic principal and eddy currents induction to simplify the visualization of defects and eddy current response